

Appendix C

Transformation Matrices

Transformation commands create the following matrices.

C.1 Translation

$$\text{Translate } (T_x, T_y, T_z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

C.2 Scaling and Mirroring

$$\text{Scale } (S_x, S_y, S_z) = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C.3 Rotation

$$\text{Rot}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}_z(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C.4 Viewing Transformations

$\text{Polarview}(\text{dist}, \text{azim}, \text{inc}, \text{twist}) = \text{Rot}_z(-\text{azim}) \cdot \text{Rot}_x(-\text{inc}) \cdot \text{Rot}_z(-\text{twist}) \cdot \text{Trans}(0.0, 0.0, -\text{dist})$
 $\text{lookat}(v_x, v_y, v_z, p_x, p_y, p_z, \text{twist}) = \text{trans}(-v_x, -v_y, -v_z) \cdot \text{rot}_y(\theta) \cdot \text{rot}_x(\phi) \cdot \text{rot}_z(-\text{twist})$

$$\sin \theta = \frac{p_x - v_x}{\sqrt{(p_x - v_x)^2 + (p_z - v_z)^2}}$$

$$\cos \theta = \frac{v_z - p_z}{\sqrt{(p_x - v_x)^2 + (p_z - v_z)^2}}$$

$$\sin \phi = \frac{v_y - p_y}{\sqrt{(p_x - v_x)^2 + (p_y - v_y)^2 + (p_z - v_z)^2}}$$

$$\cos \phi = \frac{\sqrt{(p_x - v_x)^2 + (p_z - v_z)^2}}{\sqrt{(p_x - v_x)^2 + (p_y - v_y)^2 + (p_z - v_z)^2}}$$

C.5 Perspective Transformations

$$\text{perspective}(\text{fov}, \text{aspect}, \text{near}, \text{far}) = \begin{bmatrix} \frac{\cot\left(\frac{\text{fov}}{20}\right)}{\text{aspect}} & 0 & 0 & 0 \\ 0 & \cot\left(\frac{\text{fov}}{20}\right) & 0 & 0 \\ 0 & 0 & -\frac{\text{far} + \text{near}}{\text{far} - \text{near}} & -1 \\ 0 & 0 & -\frac{2 \cdot \text{far} \cdot \text{near}}{\text{far} - \text{near}} & 0 \end{bmatrix}$$

$$\text{window}(\text{left}, \text{right}, \text{bottom}, \text{top}, \text{near}, \text{far}) = \begin{bmatrix} \frac{2 \cdot \text{near}}{\text{right} - \text{left}} & 0 & 0 & 0 \\ 0 & \frac{2 \cdot \text{near}}{\text{top} - \text{bottom}} & 0 & 0 \\ \frac{\text{right} + \text{left}}{\text{right} - \text{left}} & \frac{\text{top} + \text{bottom}}{\text{top} - \text{bottom}} & -\frac{\text{far} + \text{near}}{\text{far} - \text{near}} & -1 \\ 0 & 0 & -\frac{2 \cdot \text{far} \cdot \text{near}}{\text{far} - \text{near}} & 0 \end{bmatrix}$$

C.6 Orthographic Transformations

$$\text{ortho}(\text{left}, \text{right}, \text{bottom}, \text{top}, \text{near}, \text{far}) = \begin{bmatrix} \frac{2}{\text{right} - \text{left}} & 0 & 0 & 0 \\ 0 & \frac{2}{\text{top} - \text{bottom}} & 0 & 0 \\ 0 & 0 & -\frac{2}{\text{far} - \text{near}} & 0 \\ \frac{\text{right} + \text{left}}{\text{right} - \text{left}} & \frac{\text{top} + \text{bottom}}{\text{top} - \text{bottom}} & \frac{\text{far} + \text{near}}{\text{far} - \text{near}} & 1 \end{bmatrix}$$

$$\text{ortho2}(\text{left}, \text{right}, \text{bottom}, \text{top}) = \begin{bmatrix} \frac{2}{\text{right} - \text{left}} & 0 & 0 & 0 \\ 0 & \frac{2}{\text{top} - \text{bottom}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -\frac{\text{right} + \text{left}}{\text{right} - \text{left}} & -\frac{\text{top} + \text{bottom}}{\text{top} - \text{bottom}} & 0 & 1 \end{bmatrix}$$

